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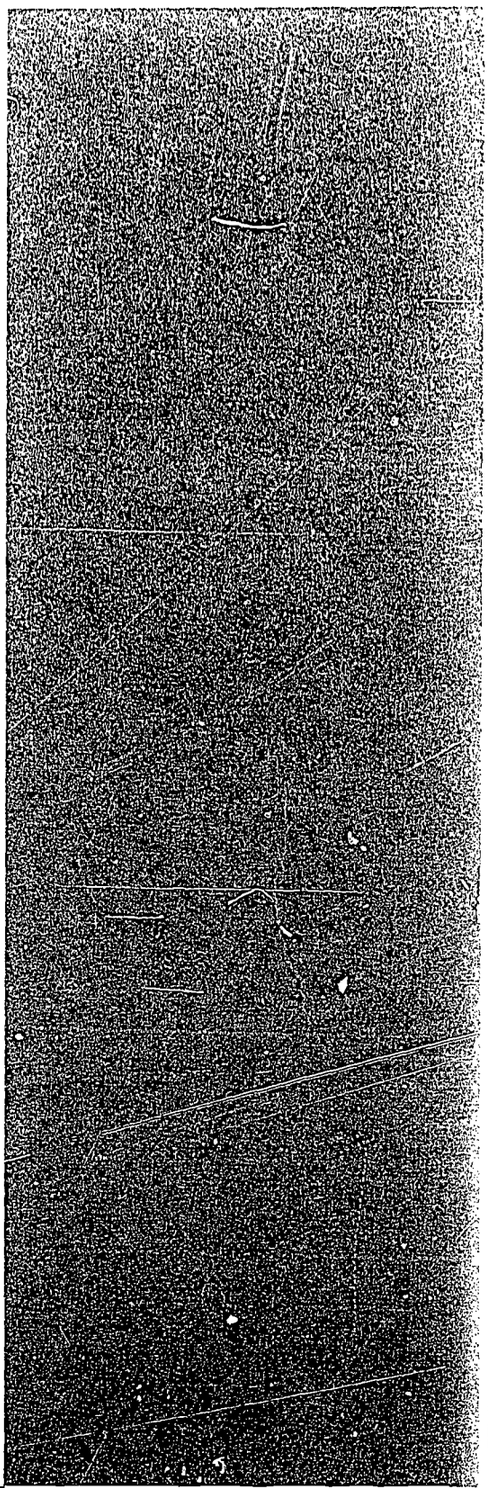
ABSTRACT

Provided are the scope and sequence of mathematics topics for Grade 6 to be implemented by teachers and supervisors. The guide is presented in the form of 62 units, each being organized around the themes of (1) sets, numbers, and numeration; (2) operations; (3) geometry and measurement; and (4) algebraic concepts, graphs, probability, and statistics. The sequence is structured to provide a spiral or cyclic approach for presentation of the concepts and skills. (Author/JG)

MATHEMATICS

• Grade 6

Scope and Sequence



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MATHEMATICS

Grade 6

Scope and Sequence

BUREAU OF CURRICULUM DEVELOPMENT
BOARD OF EDUCATION OF THE CITY OF NEW YORK

F O R E W O R D

At a time when our society is increasingly dependent upon mathematically literate citizens, it is essential that vital and contemporary mathematics be presented to our pupils. Mathematics has much to contribute to meeting the demands and the opportunities of our changing society.

Mathematics, with its structure and applications, is one of the cornerstones of a general education, which is the best insurance for helping individuals adapt to the changes of today and the needs of tomorrow.

This publication is designed to extend and strengthen the mathematical understandings and the computational skills of children in Grade 6. It is an integral part of the series, Mathematics: Scope and Sequence Grade 1; Grades 2-3; Grades 4-5 and provides an overall scope and detailed sequence for implementation by teachers and supervisors.

Sincere appreciation is expressed to the Bureau of Curriculum Development and the Bureau of Mathematics who jointly developed this bulletin. It will aid those who are concerned with teaching children to think and reason mathematically.

SEELIG LESTER
Deputy Superintendent of Schools

A C K N O W L E D G M E N T S

The preparation of this bulletin was under the general direction of Seelig Lester, Deputy Superintendent, Office of Instructional Services; David A. Abramson, Acting Director, Bureau of Curriculum Development, and George Grossman, Director, Bureau of Mathematics.

Leonard Simon, Acting Assistant Director, Bureau of Curriculum Development, supervised the project.

This material was planned and prepared by Blanche C. Gladstone, Alice D. Lombardi, and Bertha O. Weiss, Bureau of Curriculum Development.

Frank J. Wohlfort, Assistant Director, Bureau of Mathematics, assisted in evaluating the scope and sequence.

Charles J. Goode, Mathematics Coordinator, District 14, cooperated in various stages of planning and evaluation.

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Edythe Kahn, Editor, Bureau of Curriculum Development, had overall responsibility for design and production. Simon Shulman designed the cover; Ellwood White prepared it for printing.

C O N T E N T S

INTRODUCTION

The scope and sequence for Grade 6 is presented in the form of 62 units each of which may require one or more lessons.

The units are organized around four central themes. These themes are coded as shown below:

*	Sets; Number; Numeration
**	Operations
***	Geometry and Measurement
****	Algebraic Concepts; Graphs; Probability; Statistics

Units marked ⊙ are optional units and may be used at the discretion of the teacher.

The sequence is structured to provide a spiral or cyclic approach for presentation of the concepts and skills. For example, the development of division of fractional numbers begins in Unit 38, continues in Unit 39, and the understanding is reinforced and extended in Units 47 and 60.

The teacher should make provision for review, reinforcement, practice, and evaluation to meet the particular needs of the class.

Mathematical Goals for Grade 6

1. Sets; Number; Numeration

   Place value, read and write numerals through billions
   Exponential notation through 10^9
   Exponential notation with bases other than 10
   Rounding numbers to nearest million
   Roman numerals: dates, bar
   Prime and composite numbers
   Multiplicative inverse (reciprocal)
   Ratio
   Percent
   Set of integers

2. Addition and Subtraction of Whole Numbers

   Sums of two or more addends; sums beyond 1 million
   Properties of addition
   Difference of two numbers each in the millions

3. Multiplication of Whole Numbers

   Product of any whole numbers, and numbers named by 3-digit numerals
   Prime factorization
   Associative property and commutative property of multiplication; distributive property of multiplication over addition

4. Division of Whole Numbers

   Standard algorithm: divisors named by 3-digit numerals
   Tests for divisibility by 2, 5, 10, 3

5. Fractional Numbers

   Greatest common factor
   Multiplicative inverse (reciprocal)
   Division of any fractional numbers
   Ratio
   Percent
   Properties of operations

6. Fractional Numbers in Decimal Form

   Decimal system through ten-thousandths
   Expanded notation through thousandths
   Relationships among fractions, decimal fractions, percents
   Sum and difference of fractional numbers in decimal form
   Multiplication of numbers in decimal form
   Division of numbers in decimal form
   Expressing fractional numbers in decimal form; numbers in decimal form as fractional numbers

7. Geometry

Concepts: perpendicular lines in a plane, a line intersecting a plane, a line parallel to a plane, a line perpendicular to a plane, skew lines
Properties of regular polygons
Properties of trapezoid, rhombus
Concept of volume of a rectangular prism
Experimenting in geometry with ruler and protractor

8. Measurement

Linear: Circumference of a circle (about 3 times diameter)
Metric system; comparing with American units of measure
Area: Formula for computing area of rectangular region
Relationships square inches-square feet; square feet-square yards
Volume: Non-standard units
Standard unit - cubic inch
Computing volumes of rectangular solids by counting
Operations with measures
Constructing scale drawings
Using protractor to draw and measure angles

9. Algebraic Concepts

Open sentences to describe the properties of operations
Using a formula to express a rule
Graph of solutions of an equation such as $\square + \square = 8$

10. Set of Integers: Operations

Addition on number line
Commutative property of addition

11. Statistics; Probability; Graphs

Median (odd and even number of cases)
Mean
Reading, interpreting, and constructing circle graphs
Reading and interpreting scale drawings
Experimenting with tossing a die; comparing experimental results with theoretical probability

MATHEMATICS: GRADE 6

Scope and Sequence

*

1. Number; Numeration

Reinforce and/or develop place value through millions:
use a place value chart

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
10x100,000	10x10,000	10x1000	10x100	10x10	10x1	1

prepare a period chart
read and write numerals through millions
emphasize the idea that each place in a numeral has ten times the value of the place at its right

Develop understanding of exponential notation by expressing 100 as 10^2, 1000 as 10^3, etc.

1,000,000	100,000	10,000	1,000	100	10	1
10x10x10x10x10x10	10x10x10x10x10	10x10x10x10	10x10x10	10x10	10x1	1
10^6	10^5	10^4	10^3	10^2	10^1	

*

2. Number; Numeration

Reinforce understanding of exponential notation using the base 10.

Develop understanding of the use of exponents with bases other than ten. In $3^2 = 3 \times 3$ or 9; 3 is the base and 2 is the exponent.

****

3. Algebraic Concepts

Reinforce understanding of:
mathematical sentences - true, false, open
meaning of placeholder (variable)
relation symbols >, <, =, ≠
replacement set, solution set
rules for substitution
properties of addition of whole numbers

$137 + 8 \bigcirc 137 + 7$
$\square + \square = 200$
$235 + 8 = (235 + 5) + \square$
$63 + \square = 281$

Maintain skills in computing sums and differences of whole numbers by using the above concepts.

*

4. Number; Numeration

Reinforce:
rounding numbers to nearest hundred; to nearest thousand; to nearest ten-thousand; to nearest hundred-thousand.

A suggested procedure for rounding to the nearest hundred-thousand:
346,239 is between 300,000 and 400,000
350,000 is halfway between 300,000 and 400,000
346,239 < 350,000; therefore, 346,239 is nearer to 300,000
346,239 rounded to nearest hundred-thousand is 300,000

Develop rounding numbers to the nearest million.

****

5. Graphs

Use projects to provide data for constructing graphs - line, bar, picture.

Read and interpret graphs.

Note: It is recommended that graphs be used to picture data throughout the school year.

**

6. Addition and Subtraction of Whole Numbers

Continue to develop skill in computing sums and differences in vertical format, involving numbers through millions.

Estimate, compute, compare results with estimate.

Use commutative and associative properties of addition to check addition.

Use principle of related operations to check subtraction.

Include:
word problems
computations with dollars and cents

***

7. Geometry

Use pictures and diagrams to maintain and extend the following geometric ideas:

space	angles
plane	right angles
point	angles less than right angles (acute)
line segment	angles greater than right angles (obtuse)
ray	parallel lines in a plane
line	intersecting lines in a plane

Develop understanding of perpendicular lines.

****

8. Algebraic Concepts

Use open sentences and relation symbols $>$, $<$, $=$, $\neq$ to reinforce:
commutative property of multiplication
distributive property of multiplication over addition
1 as identity element of multiplication
relationship: factor × factor equals product
zero as a factor; zero as a product
use of ( ) for grouping

Suggestions illustrating these ideas are:

$24 \times 3 \bigcirc 3 \times 25$
$98 \times \square = 98$
$8 \times 32 = (8 \times 30) + (8 \times \square)$
$\square \times 21 = 0$

Reinforce and/or develop understanding that the manner of grouping three factors does not affect the product (associative property of multiplication). To find the product of $2 \times 4 \times 5$, we may group as:

$(2 \times 4) \times 5 = 8 \times 5 = 40$ or $2 \times (4 \times 5) = 2 \times 20 = 40$

Therefore, $(2 \times 4) \times 5 = 2 \times (4 \times 5)$.

Maintain skill in computing products of whole numbers by using the above concepts.

*

9. Number; Numeration

Develop understanding of divisibility; of factors.

Develop understanding of prime and composite numbers.

Develop procedure for expressing a number as a product of prime factors (prime factorization). Where possible use exponential form to record prime factorization.

*

10. Fractional Numbers

Use fractional parts, diagrams, number lines to reinforce:
- meanings of fraction
- procedures for renaming fractional numbers using the multiplicative identity (equivalent fractions)
- expressing fractional numbers in simplest form (the prime factorization method is optional)
- expressing numbers such as $\frac{12}{4}$ as 3; $\frac{13}{4}$ as $3\frac{1}{4}$; 5 as $\frac{5}{1}$; $2\frac{1}{3}$ as $\frac{7}{3}$
- understanding that between any two fractional numbers there is another fractional number

*

11. Fractional Numbers

Reinforce:
- divisibility
- common multiple
- least common multiple

Use concept of equivalent fractions and set intersection to review procedures for finding common denominator; least common denominator.

**

12. Addition of Fractional Numbers

Reinforce:
- expressing a fraction in simplest form
- renaming numbers such as $\frac{4}{2} = 2$; $\frac{9}{4} = 2\frac{1}{4}$; $\frac{10}{4} = 2\frac{2}{4} = 2\frac{1}{2}$; $12\frac{8}{3} = 14\frac{2}{3}$

Review procedures for computing sums of fractional numbers with like and unlike denominators using vertical and horizontal format. Evaluation and/or reinforcement of previous learnings should recognize such levels of difficulty as the following:

$\frac{3}{8} + \frac{2}{8} = n$

$\frac{5}{8} + \frac{1}{8} = n$

$\frac{3}{8} + \frac{5}{8} = n$

$\frac{3}{8} + \frac{6}{8} = n$

$\frac{3}{8} + \frac{7}{8} = n$

$\frac{1}{6} + \frac{1}{8} = n$

$2\frac{1}{6} + \frac{1}{8} = n$

$2\frac{1}{6} + 3\frac{1}{8} = n$

$5\frac{5}{6} + 2\frac{3}{8} = n$

$2\frac{1}{3} + 4\frac{1}{2} + 6\frac{2}{3} = n$

Observe and use commutative and associative properties for addition of fractional numbers.

Include word problems.

**

13. Subtraction of Fractional Numbers

Reinforce:
renaming numbers such as $3 = 2\frac{3}{3}$; $2\frac{3}{4} = 1\frac{7}{4}$; $3\frac{3}{4} = 3\frac{6}{8} = 2\frac{14}{8}$

Review procedures for computing differences of fractional numbers with like and unlike denominators, with and without renaming. Use horizontal and vertical format. Evaluation and/or reinforcement of previous learnings should recognize such levels of difficulty as the following:

$\frac{5}{8} - \frac{3}{8} = n$ $\qquad$ $\frac{1}{2} - \frac{1}{3} = n$

$\frac{7}{8} - \frac{1}{8} = n$ $\qquad$ $\frac{3}{5} - \frac{1}{2} = n$

$\frac{3}{8} - \frac{3}{8} = n$ $\qquad$ $3\frac{2}{3} - 1\frac{1}{4} = n$

$\frac{9}{8} - \frac{1}{8} = n$ $\qquad$ $7 - \frac{3}{5} = n$

$1\frac{3}{8} - \frac{3}{8} = n$ $\qquad$ $5\frac{1}{3} - 2\frac{4}{5} = n$

$3\frac{1}{8} - \frac{2}{8} = n$ $\qquad$ $5\frac{1}{3} - 2\frac{4}{5} = n$

$2\frac{5}{8} - \frac{7}{8} = n$ $\qquad$ $27\frac{1}{2} - 13\frac{2}{3} = n$

Check solutions by using the principle of related operations.

Include word problems.

****

14. Algebraic Concepts

Use experiences to reinforce and/or develop the understanding of:
graph of a number on the number line
graph of a solution set of an open sentence on a number line
an ordered pair of numbers (5,2) and the procedures for associating an ordered pair of numbers with a point
procedures for graphing solutions of an equation such as $\square + \nabla = 8$

*

15. Number; Numeration: Decimals

Reinforce:
understanding of multiplicative relationships developed in Unit 1
understanding that each place in our numeration system has a value $\frac{1}{10}$ the value of the place at its left
understanding of tenths and hundredths in decimal form

Extend understanding of the numeration system to thousandths:
prepare place value chart
record in various ways, e.g., $\frac{16}{1000}$, .016, 16 thousandths
read and write numerals involving thousandths
write tenths as hundredths and as thousandths
rename numbers such as 1.2 as 12 tenths, 46.3 as 463 tenths
compare numbers, e.g., .006 < .06; .4 > .009
express numbers in expanded notation, e.g., .647 = 6 tenths + 4 hundredths + 7 thousandths or as .6 + .04 + .007

**

16. Addition and Subtraction of Numbers in Decimal Form

Reinforce:
reading and writing decimal numerals in tenths, hundredths, thousandths
understanding of the decimal system by using number series such as:

3.1, ____, ____, 3.4, 3.5
4.18, 4.19, ____, ____, 4.22
5.008, 5.009, ____, ____, 5.012
7.0, ____, ____, 7.3, 7.4

procedures for computing sums and differences using tenths and hundredths in decimal form

Develop procedures for computing sums and differences of numbers in the thousandths in decimal form.

Estimate, compute, compare results with estimate.

Use commutative and associative properties of addition.

Use principle of related operations to check subtraction.

Include word problems.

**

17. Operations with Measures

Reinforce:
converting to different units of linear measure; liquid measure; weight; time
computing fractional parts of a unit of measure: ½ mile = □ feet

Develop procedures for operating with measurements:

6 ft. 7 in. + 1 ft. 9 in. = □
4 hr. 10 min. - 1 hr. 40 min. = □
2 x (3 yd. 2 ft.) = □
3 lb. 12 oz. ÷ 3 = □

***

18. Measurement: Circumference

Reinforce:
meaning of a circle
understanding that a circle divides a plane into three sets of points
understanding of line segments in a circle: radius, diameter, chord
understanding of circumference

Use projects to discover that the circumference is about three times the length of the diameter.

**

19. Multiplication of Whole Numbers

Reinforce multiplying any whole numbers by numbers in the tens, e.g.,

$$\begin{array}{r} 514 \\ \times 32 \\ \hline \end{array}$$

Continue to develop procedures for multiplying numbers in the hundreds by numbers named by 3-digit numerals selected from digits 0,1,2,3,4.5.

$$\begin{array}{r} 514 \\ \times 332 \\ \hline \end{array}$$

**

20. Division of Whole Numbers

Reinforce dividing whole numbers by numbers less than 100.

Develop division with:

Divisors: numbers named by 3-digit numerals selected from the digits 0,1,2,3, e.g., 221
Dividends: numbers in thousands

**

21. Multiplication of Fractional Numbers

Recall the generalization: to find the product of two fractional numbers multiply the numerators and multiply the denominators.

Reinforce computing products recognizing such levels of difficulty as:

$\frac{1}{2} \times \frac{3}{4} = n$ $\qquad$ $\frac{1}{2} \times 2\frac{1}{4} = n$

$3 \times \frac{3}{10} = n$ $\qquad$ $1\frac{3}{4} \times 2\frac{1}{2} = n$

Use and observe the properties of multiplication of fractional numbers.

**

22. Multiplication of Numbers in Decimal Form

Reinforce renaming:

.3 as $\frac{3}{10}$ as 3 tenths

1.2 as $1\frac{2}{10}$ as $\frac{12}{10}$ as 12 tenths

$\frac{9}{10}$ as .9

$\frac{12}{10}$ as 1.2

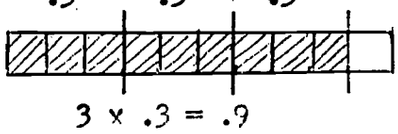
9 tenths as .9

12 tenths as 1.2

Develop generalization for computing the product of a whole number and tenths in decimal form.

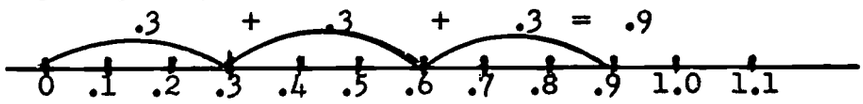
Suggested procedures to compute $3 \times .3 = .9$ or $\begin{array}{r} .3 \\ \times 3 \\ \hline .9 \end{array}$

Diagram:



$3 \times .3 = .9$

Number line:



$3 \times .3 = .9$

Fraction form: $3 \times \frac{3}{10} = \frac{9}{10}$ or .9

Word name: 3 x 3 tenths = 9 tenths or .9

In a similar manner develop:

$7 \times .3 = n$
$3 \times 1.3 = n$

Use commutative property of multiplication.

***

23. Geometry

Reinforce understanding of:
- lines in a plane
- intersecting lines
- perpendicular lines
- parallel lines
- two intersecting planes, two parallel planes

Develop understanding of a line intersecting a plane; of a line perpendicular to a plane; of a line parallel to a plane; of skew lines.

**

24. Division of Numbers in Decimal Form

Reinforce:
- renaming .3 as 3 tenths
  1.2 as 12 tenths
  24.9 as 249 tenths
  3 tenths as .3
  12 tenths as 1.2
- rounding tenths to the nearest whole number

Develop generalization for computing the quotient of tenths in decimal form by a whole number.

Use experiences, verbal problems, diagrams, number lines, word names to compute $.8 \div 2$ or $2\overline{).8}$. (Avoid problems such as $.4 \div 5$, $3\overline{).2}$.)

Use word names:

$.8 \div 2$ or $2\overline{).8}$ can be expressed as $\frac{4 \text{ tenths}}{2\overline{)8 \text{ tenths}}}$ or .4

In a similar manner develop:

$6.3 \div 3 = n$ $\qquad$ $1.2 \div 4 = n$

Estimate, compute, check.

$24.4 \div 4 = n$
24.4 is rounded to 24
$24 \div 4 = 6$
$24.4 \div 4$ is about 6

$$\begin{array}{r} 6.1 \\ 4\overline{)24.4} \end{array}$$

Check: $\begin{array}{r} 6.1 \\ \times\ 4 \\ \hline 24.4 \end{array}$

*

25. Set of Integers

Recall meaning of the set of whole numbers; set of counting numbers; set of fractional numbers.

Use horizontal and vertical number lines and games to develop the meaning of the set of integers.

Include:
- positive and negative direction from zero
- symbols for positive and negative numbers: $^{+}1$; $^{-}1$.
- meaning of opposites
- order in the set of integers
- comparisons using >, <; $^{+}3 > 0$; $^{-}2 < 0$

**

26. Multiplication of Numbers in Decimal Form

Adapt suggestions in Unit 22 to develop generalization for computing the product of a whole number and hundredths in decimal form; of a whole number and thousandths in decimal form.

**

27. Division of Numbers in Decimal Form

Adapt suggestions in Unit 24 to develop generalization for computing the quotient of hundredths in decimal form by a whole number; of thousandths in decimal form by a whole number.

***

28. Measurement: Metric System

Use materials such as meter stick, yardstick, metric desk rulers to reinforce and/or develop understanding of the metric system of linear measurement.

Develop relationships:
- within metric system through millimeter and kilometer
- meter-yard; centimeter-inch; kilometer-mile

Provide problems using metric units of measure.

*

29. Number; Numeration

Adapt suggestions in Unit 1 to develop an understanding of place value and exponential notation through billions.

*

⊙30. Numeration: Bases Other Than 10

Experiment with materials such as discs, squared materials, multibase blocks to discover that there are various ways of grouping objects which result in different numerals that name the same number, e.g., thirteen discs may be grouped as four groups of three and one one; as two groups of five and three ones; as one group of seven and six ones; as one group of ten and three ones.

Reinforce some characteristics of the decimal system of numeration such as:

10 basic symbols or digits - {0,1,2,3,4,5,6,7,8,9}
place value

Reinforce and/or develop an understanding of the base 5 system of numeration.

Include:

grouping by fives
basic symbols in base five {0,1,2,3,4}
place value chart for base five
interpreting numerals in base five by means of place value chart
comparing numerals in base five with numerals in base ten

**

31. Addition and Subtraction of Whole Numbers

Adapt suggestions indicated in Unit 6 to continue to develop skill in computing sums and differences, in vertical format, involving any whole numbers.

Note: It is recommended that to maintain skill, practice in computing sums and differences be provided throughout the school year.

**

32. Multiplication of Whole Numbers

Continue to develop procedures for multiplying whole numbers by numbers named by any 3-digit numeral.

Note: It is recommended that to maintain skill, practice in computing products be provided throughout the school year.

**

33. Division of Whole Numbers

Continue to develop dividing whole numbers by numbers named by any 3-digit numeral.

Note: It is recommended that to maintain skill, practice in computing quotients be provided throughout the school year.

****

34. Graphs; Statistics

Use projects involving experiences to reinforce understanding that:
- the mode of a set of scores is the one that occurs most frequently
- the mean (average) of a set of scores is the sum of all the scores divided by the total number of scores
- the median of a set of an odd number of scores arranged in order of size is the middle score

Develop the understanding that in an even number of scores the median is the average of the two middle scores.

*

35. Numeration: Roman Numerals

Reinforce:
- repetitive, additive, and subtractive principles of the Roman system of numeration
- understanding of the symbols: M, D, C, L, X, V, I

Use Roman numerals to record dates such as birth dates, significant dates from other subject areas, etc.

Develop understanding of the use of the bar to show multiplication by 1000, e.g., $\overline{VI}$ names 6000.

*

36. Fractional Numbers

Reinforce:
- expressing a fraction in simplest form by inspection; by finding a common factor
- expressing a number as a product of prime factors (prime factorization)

Develop a procedure for finding the greatest common factor. Use these understandings to express fractions in simplest form.

***

37. Geometry and Measurement

Reinforce:
- meaning of polygon
- understanding that a polygon divides a plane into three sets of points
- properties of regular polygons

Experiment with models of trapezoid and rhombus to discover some of their characteristics.

Reinforce addition and multiplication of whole numbers, of fractional numbers, of numbers in decimal form by computing perimeters of polygons.

Use problems involving units such as 3 feet 1 inch, as well as metric units.

*

38. Number

Reinforce the understanding that the result of dividing a number by one is the number itself.

$318 \div 1 = \square$ $\qquad$ $\frac{3}{5} \div 1 = \square$

(division property of 1)

Develop the understanding that if the product of two numbers is one, each number is the reciprocal of the other.

$\frac{3}{4} \times \frac{4}{3} = \frac{12}{12}$ or 1	$\frac{3}{4}$ is the reciprocal of $\frac{4}{3}$ and $\frac{4}{3}$ is the reciprocal of $\frac{3}{4}$
$5 \times \frac{1}{5} = \frac{5}{1}$ or 1	5 is the reciprocal of $\frac{1}{5}$ and $\frac{1}{5}$ is the reciprocal of 5
$3\frac{1}{2}$ may be expressed as $\frac{7}{2}$	
$\frac{7}{2} \times \frac{2}{7} = \frac{14}{14}$ or 1	$\frac{7}{2}$ is the reciprocal of $\frac{2}{7}$ and $\frac{2}{7}$ is the reciprocal of $\frac{7}{2}$

**

39. Division of Fractional Numbers

Experiment with division to show that multiplying dividend and divisor by the same non-zero number does not change the quotient. Consider

$8 \div 2 = 4$

Does $(8 \times 2) \div (2 \times 2) = 4$?	Dividend and divisor are both multiplied by 2
$16 \div 4 = 4$	The quotient is not changed

Consider $8 \div 2 = 4$

Does $(8 \times \frac{1}{2}) \div (2 \times \frac{1}{2}) = 4$? — Dividend and divisor are both multiplied by $\frac{1}{2}$

$4 \div 1 = 4$ — The quotient is not changed

Develop procedures for dividing a fractional number by a fractional number using:
- division property of 1 (Unit 38)
- reciprocal (Unit 38)
- rule that multiplying dividend and divisor by the same number does not change the quotient

Consider:

$\frac{2}{5} \div \frac{3}{4} = \square$ — If the divisor were 1, the quotient could easily be found

$(\frac{2}{5} \times \frac{4}{3}) \div (\frac{3}{4} \times \frac{4}{3}) = \square$ — Multiplying dividend and divisor by the same number does not change the quotient. For a divisor of 1, select the reciprocal of the divisor as the multiplier.

$\frac{8}{15} \div \frac{12}{12} = \square$

$\frac{8}{15} \div 1 = \frac{8}{15}$

Note: At this time, use only fractional numbers that are less than 1.

**

40. Addition in the Set of Integers

Reinforce understanding of set of integers. (Unit 25)

Use games and the number line to add two positive integers; two negative integers.

Observe commutative property of addition.

*

41. Number; Numeration: Decimals

Adapt suggestions in Unit 15 to extend understanding of the numeration system to ten-thousandths.

**

42. Addition and Subtraction of Numbers in Decimal Form

Adapt suggestions in Unit 16 to develop procedures for computing sums and differences of numbers in the ten-thousandths in decimal form.

Note: It is recommended that to maintain skill, practice in computing sums and differences be provided throughout the school year.

**

43. Multiplication and Division of Numbers in Decimal Form

Adapt suggestions in Units 22 and 26 to develop generalization for computing the product of a whole number and ten-thousandths in decimal form.

Adapt suggestions in Units 24 and 27 to develop generalization for computing the quotient of ten-thousandths in decimal form by a whole number.

*

44. Number: Ratio

Use experiences to develop the understanding of ratio.

A ratio may be expressed in various ways such as:

x x x x
□□□□□□

The ratio of the number of x's to the number of □'s is:

4 to 6 4:6 $\frac{4}{6}$ $\frac{2}{3}$

▨▨□□□

The ratio of the number of ▨'s to the total number is:

2 to 5 2 out of 5 2:5 $\frac{2}{5}$

***

45. Measurement: Scale Drawing

Use projects to reinforce constructing and interpreting graphs - line, bar.

Use materials such as maps and diagrams to practice interpreting drawings made to scale.

Through projects develop procedures for making drawings to scale.

***

46. Measurement: Area

Reinforce:
- meaning of rectangular region
- understanding that the measure of a region is called area
- relationships: square inches-square feet; square feet-square yards

Use materials to develop formula for computing area of a rectangular region.

**

Unit 47. Division of Fractional Numbers

Adapt suggestions in Unit 39 to develop procedures for dividing a fractional number by a whole number; a whole number by a fractional number.

****

Unit 48. Algebraic Concepts

Reinforce:
understanding of positive and negative numbers (set of integers)
graph of positive and negative numbers on horizontal and vertical number lines

Review procedures for associating an ordered pair of numbers with a point in a plane.

Develop procedures for graphing solutions of an equation of the form $\Diamond + \square = 5$ in a plane.

$\Diamond$	$\square$
$+1$	$+4$
-3	$+8$

**

Unit 49. Multiplication of Numbers in Decimal Form

Adapt suggestions in Unit 22 to develop generalization for computing the product of tenths in decimal form.
Include the following:

$.3 \times .5 = n$ $.3 \times .2 = n$ $1.7 \times .3 = n$ $2.5 \times 1.3 = n$

**

Unit 50. Division of Numbers in Decimal Form

Reinforce multiplying whole numbers by 10.

Develop generalization for multiplying numbers in decimal form by 10.

$10 \times .9 = \square$ $10 \times 1.9 = \square$
$10 \times .09 = \square$ $10 \times 23.9 = \square$
$10 \times .009 = \square$ $10 \times 23.09 = \square$

Recall that multiplying dividend and divisor by the same non-zero number does not change the quotient (Unit 39).

Recall how to divide a number in decimal form by a whole number.

Develop generalization for computing the quotient when the divisor is tenths in decimal form.

Consider	$.3\overline{).9}$	If the divisor were a whole number, the quotient could easily be found.
	$10 \times .3 = 3$	One way to obtain a whole number divisor is to multiply by 10.
	$3\overline{)9}$	Multiply divisor and dividend by 10 and proceed with the division.

Use a similar procedure to compute quotients such as:

$.3\overline{)5.1}$ $.4\overline{).08}$ $.4\overline{)1.88}$ $1.1\overline{)13.2}$ $.5\overline{)49}$

***

51. Measurement: Volume

Use physical models such as shoebox, cereal box, rectangular block to introduce idea of rectangular prism.

Review properties of rectangular prisms.

Develop the understanding that a rectangular prism separates space into three sets of points:
the set of points inside the prism (interior)
the set of points on the prism
the set of points outside the prism (exterior)

Present the concept that the measure of the enclosed space is called volume.

Experiment with a variety of ways to determine the volume of a rectangular prism and to discover the need for a standard unit of measure such as cubic inch.

Use materials to develop procedures for determining the volume of rectangular prisms by counting the number of cubic inches contained in each.

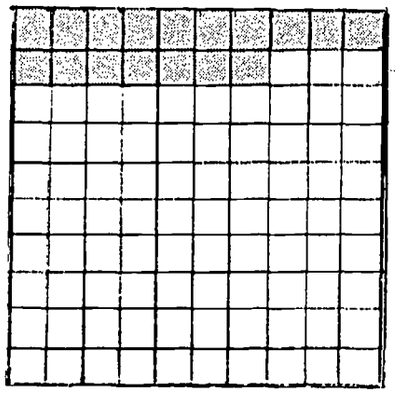
*

52. Percent

Reinforce understanding of ratio.

Use activities with squared materials, graph paper to develop understand-

ing that one way of expressing the ratio of a number to 100 is as a percent.



17 out of the 100 squares are shaded

$\frac{17}{100}$ is shaded

17% is shaded

*
⊙53. Modulo Number System

Reinforce understanding of clock arithmetic. (12 hour clock)
Observe properties of addition.

Reinforce and/or develop understanding of modulo systems such as 7, 24.
Addition in modulo system.
Properties of addition in modulo system.

**
54. Multiplication of Numbers in Decimal Form

Adapt suggestions in Units 22 and 49 to develop generalization for computing the product of numbers in decimal form - not more than four decimal places in the product.

*
55. Percent

Reinforce understanding of percent.

Develop the understanding that percents, fractions, and decimals are ways of expressing ratio.

3%	$\frac{3}{100}$	.03

Include percents that can easily be renamed.

50%	$\frac{50}{100}$ or $\frac{1}{2}$	.50
75%	$\frac{75}{100}$ or $\frac{3}{4}$	.75
40%	$\frac{40}{100}$ or $\frac{2}{5}$	.40

***

56. Geometry and Measurement

Reinforce understanding of angle and central angle.

Introduce the use of protractor to draw and measure angles.

Experiment with ruler and protractor. Activities may include:
draw a rectangle with sides 2 inches x 3 inches
draw a right triangle with legs 3 inches and 4 inches
draw a triangle with sides of the same length

****

57. Graphs

Read and interpret circle graphs.

Reinforce:
understanding of central angle
use of protractor to draw and measure angles

Develop procedures for constructing circle graphs.

**

58. Division of Numbers in Decimal Form

Adapt suggestions in Unit 50 to develop generalization for computing the quotient when the divisor is hundredths in decimal form; thousandths in decimal form.

**

59. Division of Whole Numbers

Reinforce the meaning of divisibility.

Discover tests for divisibility by 2, 5, 10, 3.

**

60. Division of Fractional Numbers

Adapt suggestions in Units 39 and 47 to develop procedures for dividing any fractional numbers.

$$1\frac{1}{2} \div 1\frac{2}{3} = n$$

****

61. Probability

Experiment with tossing one die.

Tally, tabulate, and graph results after 6 throws, 18 throws, 30 throws.

Compare predicted with actual results.

**

⊙62. Addition in the Set of Integers

Reinforce adding two positive integers; two negative integers.

Use games and number line to add a positive and a negative integer.

Use and observe commutative property of addition.